

# THE LOCAL STABILITY OF ACCRETION DISKS WITH ADVECTION

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## Abstract

Based on the discussion of the applicability of local approximation, the local stability of accretion disks with advection is studied together with the considerations of radial viscous force and thermal diffusion. For a geometrically thin, radiative cooling dominated disk, the thermal diffusion has nearly no effects on the thermal and viscous modes, which are both stable if the disk is also optically thick, gas pressure dominated and are both unstable if the disk is whether optically thick, radiation pressure dominated or optically thin. The including of thermal diffusion, however, tends to stabilize the acoustic modes which, if without advection, are unstable if the disk is optically thick, radiation pressure dominated or optically thin, and are stable if the disk is optically thick, gas pressure dominated. The including of very little advection has significant effects on two acoustic modes, which are no longer complex conjugates each other. Independent on the optical

depth, the instability of the outward propagating mode (O-mode) is enhanced and that of the inward propagating mode (I-mode) is damped if the disk is gas pressure dominated, while the instability of O-mode is damped and that of I-mode is enhanced if the disk is radiation pressure dominated. For a geometrically slim, advection-dominated disk, both the thermal and viscous modes, as well as I-mode, are always stable if the disk is optically thin. The including of thermal diffusion tends to make these modes more stable. However, the O-mode can become unstable when  $q/m$  is very large ( $q$  is the ratio of advective to viscous dissipated energy and  $m$  the Mach number), even if the thermal diffusion is considered. On the other hand, if the advection-dominated disk is optically thick, we found there is no self-consistent acoustic modes in our local analyses. The thermal diffusion has no effect on the stable viscous mode but has a significant contribution to enhance the thermal instability.

*Subject headings:* Accretion, accretion disks - instabilities

## 1. Introduction

The stability of geometrically thin accretion disks has been extensively studied after the construction of standard  $\alpha$  model (Shakura & Sunyaev 1973). It has been found that the disk is thermally and viscously unstable if it is optically thick and radiation pressure dominated (Pringle, Rees & Pacholczyk 1973; Lightman & Eardley 1976; Shakura & Sunyaev 1976). Subsequent studies have found that the optically thick disk admits not only the thermal and viscous instabilities but also the inertial-acoustic (or pulsational) instability (Kato 1978; Blumenthal, Yang & Lin 1984). If the geometrically thin disk is optically thin, it has been also found it is viscously stable but thermally unstable (Piran 1978). Those instabilities are believed to be relevant to some light variations observed in many systems such as cataclysmic variables, X-ray binaries and active galactic nucleus. For example, the thermal instability may account for the periodic outburst of dwarf nova (Osaki 1974) and the inertial-acoustic instability may explain the observed QPO phenomena in Galactic black hole candidates (Chen & Taam 1995).

In the standard  $\alpha$  model, the viscous heating balances by radiative cooling. However, if the radiative cooling is not efficient, the advection will be not negligible. Particularly in an optically thin disk, the radiative cooling rate is so low that most of the viscous generated energy is advected radially. Recently, the accretion disk models with advection have been studied when the disk is either optically thick or optically thin ( Abramowicz et al. 1988; Kato, Honma & Matsumoto 1988; Narayan & Popham 1993; Narayan & Yi 1994, 1995a,b; Abramowicz et al. 1995; Chen et al. 1995; Chen 1995). The advection-dominated disk model has also been successfully adopted to explain the observations of both low luminosity and high luminosity systems (Narayan, Yi & Mahadevan 1995; Narayan, McClintock & Yi 1996; Lasota et al 1996; Narayan 1996).

Although some of the stability properties of accretion disks with advection has been

suggested in the previous research by analysing the disk structure, the detailed stability analysis has not yet been well done. From the  $\dot{M} - \Sigma$  relation, the stability properties of thermal and viscous modes in accretion disks with advection can be obtained by analysing the  $\dot{M}(\Sigma)$  slope and comparing the cooling and heating rates near each equilibrium curve. In particular, the advection-dominated disks are suggested to be both thermal and viscous stable whether the disks are optically thin or optically thick (Chen et al 1995). However, such a stability analysis can only be applied to the long-wavelength perturbations (Chen 1996). In order to confirm the previous results, the detailed stability analyses of accretion disks should be done by considering the perturbations to the time-dependent equations. More recently, Kato, Abramowicz & Chen (1996) performed an analytic stability analysis to the advection-dominated disks by considering the local perturbations. They found the optically thick disks are still thermally unstable when the thermal diffusion is considered, while the optically thin disks are always stable. Furthermore, they thought that in the case of an advection-dominated disk, the variations of angular momentum and of surface density associated with the perturbations lead to thermal instability, which is quite different from that in a radiative cooling dominated disk where there is no appreciable surface density change (Pringle 1976).

Stimulated by the above research, we have performed a detailed study to the local stability of accretion disks with advection. The influences of thermal diffusion on the disk instability were also considered. Different from the study of Kato et al. (1996), we discussed not only the stability of thermal mode, but also the viscous and inertial-acoustic modes. Moreover, a general dispersion relation, which is suitable for the stability analysis of accretion disks with different disk structures, was obtained. In Section 2 we give the basic equations and a discussion about the validity of local approximation. In Section 3 we derive the perturbed equations and the dispersion relation. The stability properties of

optically thin and optically thick disks are presented respectively in Section 4 and Section 5. Finally in Section 6, a brief discussion about our results is given.

## 2. Basic equations and local approximation

We consider an axisymmetric and non-self-gravitating accretion disk. The effects of general relativity are introduced by the pseudo-Newtonian potential (Paczynski & Wiita 1980),  $\Psi = -GM/(R - r_g)$ , where  $M$  is the mass of central object,  $R = (r^2 + z^2)^{1/2}$  and  $r_g = 2GM/c^2$ . Adopting a cylindrical system of coordinates  $(r, \varphi, z)$  which is centered on the central object, the vertical integrated time-dependent equations describing accretion flow can be written as:

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(r \Sigma V_r) = 0, \quad (1)$$

$$\Sigma \frac{\partial V_r}{\partial t} + \Sigma V_r \frac{\partial V_r}{\partial r} - \Sigma(\Omega^2 - \Omega_k^2)r = -2 \frac{\partial(Hp)}{\partial r} + F_\nu, \quad (2)$$

$$\Sigma r^3 \frac{\partial \Omega}{\partial t} + \Sigma r V_r \frac{\partial(r^2 \Omega)}{\partial r} = \frac{\partial}{\partial r}(\Sigma \nu r^3 \frac{\partial \Omega}{\partial r}), \quad (3)$$

$$C_v[\Sigma \frac{\partial T}{\partial t} + \Sigma V_r \frac{\partial T}{\partial r} - (\Gamma_3 - 1)T(\frac{\partial \Sigma}{\partial t} + V_r \frac{\partial \Sigma}{\partial r})] = \Sigma \nu (r \frac{\partial \Omega}{\partial r})^2 - Q_- + Q_+, \quad (4)$$

where  $V_r$ ,  $\Omega$  are the radial velocity and angular velocity,  $p$ ,  $T$  and  $\Sigma$  are the total pressure, temperature and surface density,  $C_v$  and  $\Gamma_3$  are the heat capacity per unit mass and a quantity associated with  $\beta$ , the ratio of gas to total pressure. The total pressure  $p$  is the sum of gas and radiation pressures given by  $p = \Re \rho T + aT^4$ , where  $\Re$  is the gas constant and  $a$  the radiation constant.  $\Omega_k$  is the Keplerian angular velocity, given by  $\Omega_k^2 = (\frac{\partial \Psi}{r \partial r})_{z=0}$ .  $F_\nu$  is the radial viscous force, which is often neglected in geometrically thin accretion disks but is perhaps not negligible in accretion disks with advection. It is given by (Papaloizou & Stanley 1986)

$$F_\nu = \frac{\partial}{\partial r}[\frac{1}{2} \frac{\nu_r \Sigma}{r} \frac{\partial(r V_r)}{\partial r}] - \frac{2 V_r}{r} \frac{\partial(\nu_r \Sigma)}{\partial r}, \quad (5)$$

where  $\nu_r$  is the kinematic viscosity acting in the radial direction. In this paper we take  $\nu_r = \nu$ , where  $\nu$  is the viscosity acting in the azimuthal direction and is expressed as the standard  $\alpha$  prescription (Shakura & Sunyaev 1973),  $\nu = \alpha c_s H$ .  $c_s$  is the local sound speed defined by  $c_s = p/\rho$ , where  $\rho$  is the density.  $H$  is the disk height given by  $H = c_s/\Omega_k$ .  $Q_-$  at the right side of Eq. (4) represents the radiative cooling. For an optically thick disk, we take it as

$$Q_- = \frac{8acT^4}{3\kappa\Sigma}, \quad (6)$$

where  $\kappa$  is the opacity given by electron scattering:  $\kappa_{es} = 0.34$ . For an optically thin disk, we assume  $Q_-$  is provided by thermal bremsstrahlung with emissivity ( $\text{ergs}^{-1}\text{cm}^{-2}$ )

$$Q_- = 1.24 \times 10^{21} H \rho^2 T^{1/2} A, \quad (7)$$

where  $A \geq 1$  is the Compton enhancement factor.  $Q_t$  at the right side of Eq. (4) represents the thermal diffusion defined as  $Q_t = \nabla \cdot (K \nabla T)$ , where  $K$  is the vertical integrated thermal conductivity given by  $K = \Sigma C_v \nu = \alpha f \Omega_k H^3 p / T$ , where  $f = 3(8 - 7\beta)f_*$  and  $f_*$  is of the order of unity (Kato et al. 1996).

In this paper we will consider the local perturbations to the accretion disks. The radial perturbations of  $V_r, \Omega, \Sigma$  and  $T$  are of the form  $(\delta V_r, \delta \Omega, \delta \Sigma, \delta T) \sim e^{i(\omega t - kr)}$ , where  $k$  is the perturbation wavenumber defined by  $k = 2\pi/\lambda$ ,  $\lambda$  is the perturbation wavelength. The local approximation means  $\lambda < r$  and the validity of the vertically integrated equations requires  $kV_r < \Omega_k$ . Since  $V_r \sim \alpha c_s^2 / \Omega_k$ , the requirements above can be written as

$$\frac{r}{H} > \frac{\lambda}{H} > 2\pi\alpha \frac{H}{r}. \quad (8)$$

We can see clearly that this inequality is well satisfied for a geometrically thin accretion disk even if we set  $\lambda/H$  in a wide range, such as from 1 to 100. However, for a geometrically slim disk, where  $H/r \leq 1$ , it can be satisfied only when  $\alpha$  is sufficiently small. The range of  $\lambda/H$  also moves to the smaller value such as from 0.06 to 1 if  $\alpha$  is about 0.01.

In addition, the validity of vertical integrated equations also requires the growth rates of unstable modes are less than the angular velocity (Kato et al. 1996). In order to get reasonable and self-consistent results, we present our discussions in this paper following all these restrictions.

### 3. Perturbed equations and dispersion relation

Considering the radial perturbations to  $V_r, \Omega, \Sigma, T$  and the local approximation, the perturbed equations corresponding to Eqs.(1)-(4) can be written as followings after a lengthy deduction.

$$\tilde{\sigma} \frac{\delta \Sigma}{\Sigma} - i \frac{\epsilon}{\tilde{H}} \frac{\delta V_r}{\Omega_k r} = 0, \quad (9)$$

$$-i\epsilon \tilde{H} \frac{\delta \Sigma}{\Sigma} + (\tilde{\sigma} + \frac{4}{3}\alpha\epsilon^2) \frac{\delta V_r}{\Omega_k r} - 2\tilde{\Omega} \frac{\delta \Omega}{\Omega_k} - ix_1 \epsilon \tilde{H} \frac{\delta T}{T} = 0, \quad (10)$$

$$ix_2 \alpha \epsilon \tilde{H} g \frac{\delta \Sigma}{\Sigma} + \frac{\tilde{\chi}^2}{2\tilde{\Omega}} \frac{\delta V_r}{\Omega_k r} + (\tilde{\sigma} + \alpha\epsilon^2) \frac{\delta \Omega}{\Omega_k} + ix_3 \alpha \epsilon \tilde{H} g \frac{\delta T}{T} = 0, \quad (11)$$

$$-(y_1 \tilde{\sigma} + \alpha y_2 g^2) \frac{\delta \Sigma}{\Sigma} - \frac{\alpha q g^2}{m \tilde{H}} \frac{\delta V_r}{\Omega_k r} + \frac{2i\alpha \epsilon g}{\tilde{H}} \frac{\delta \Omega}{\Omega_k} + (y_4 \tilde{\sigma} - \alpha x_2 g^2 + \alpha f \epsilon^2) \frac{\delta T}{T} = 0, \quad (12)$$

where  $\tilde{\sigma} = \sigma/\Omega_k$ , and  $\sigma = i(\omega - kV_r)$ .  $\tilde{\Omega} = \Omega/\Omega_k$ ,  $\tilde{H} = H/r$ , and  $\epsilon = kH$ .  $g = \frac{\tilde{\chi}^2}{2\tilde{\Omega}} - 2\tilde{\Omega}$ , and  $\tilde{\chi} = \chi/\Omega_k$  where  $\chi$  is the epicyclic frequency defined by  $\chi^2 = 2\Omega(2\Omega + r\frac{\partial\Omega}{\partial r})$ .  $m$  is the Mach number defined by  $m = |V_r|/c_s$ .  $q$  is the ratio of advective energy to viscous dissipated energy, namely

$$C_v [\Sigma V_r \frac{\partial T}{\partial r} - (\Gamma_3 - 1)T(\frac{\partial \Sigma}{\partial t} + V_r \frac{\partial \Sigma}{\partial r})] = q \Sigma \nu (r \frac{\partial \Omega}{\partial r})^2. \quad (13)$$

If the disk is radiative cooling dominated,  $q$  is nearly zero and if it is advection-dominated,  $q$  is nearly 1. The values of  $x_1, x_2, x_3, y_1, y_2, y_3$  and  $y_4$  in the perturbed equations depend on the structure properties of accretion disks. For a gas pressure dominated disk,  $x_1 = x_2 = x_3 = 1$ , and  $y_1 = 1, y_2 = 1 - 2q, y_3 = \frac{3}{2}, y_4 = -1$  if it is optically thin while  $y_1 = 1, y_2 = 2 - q, y_3 = \frac{3}{2}, y_4 = 3 - 4q$  if it is optically thick. For a radiation pressure

dominated disk,  $x_1 = -8$ ,  $x_2 = -1$ ,  $x_3 = 8$ , and  $y_1 = 4$ ,  $y_2 = -q$ ,  $y_3 = 12$ ,  $y_4 = -4(1 + q)$  if it is optically thick.

By setting the determinants of the coefficients in above perturbed equations to zero, we get a dispersion relation:

$$a_1\tilde{\sigma}^4 + a_2\tilde{\sigma}^3 + a_3\tilde{\sigma}^2 + a_4\tilde{\sigma} + a_5 = 0, \quad (14)$$

where  $a_i (i = 1, \dots, 5)$  is the coefficients given by

$$\begin{aligned} a_1 &= y_3, \\ a_2 &= \alpha[\epsilon^2(f + \frac{7}{3}y_3) + y_4g^2], \\ a_3 &= \alpha\epsilon g^2[\alpha\epsilon(\frac{7}{3}y_4 + 2x_3) - ix_1\frac{q}{m}] + \frac{1}{3}(\alpha\epsilon^2)^2(4y_3 + 7f) + \epsilon^2(y_3 + x_1y_1) + y_3\tilde{\chi}^2, \\ a_4 &= 2ix_3\alpha^2\tilde{\Omega}\epsilon g^3\frac{q}{m} + \alpha g^2[\frac{4}{3}y_4(\alpha\epsilon^2)^2 - ix_1\alpha\epsilon^3\frac{q}{m} + \frac{8}{3}x_3(\alpha\epsilon^2)^2 + y_4\tilde{\chi}^2 + (y_4 + x_1y_2)\epsilon^2] + \\ &\quad \alpha\epsilon^2 g[2x_1\frac{\tilde{\chi}^2}{2\tilde{\Omega}} - 2\tilde{\Omega}(y_1x_3 + x_2y_3)] + \alpha\epsilon^2[\frac{4}{3}f(\alpha\epsilon^2)^2 + f\tilde{\chi}^2 + \epsilon^2(y_3 + f + x_1y_1)], \\ a_5 &= (\alpha\epsilon)^2[-2\tilde{\Omega}g^3(x_3y_2 + x_2y_4) + \epsilon^2g^2(-2x_1x_2 + x_1y_2 + 2x_3) + \epsilon^2g(y_4 - 2x_2f\tilde{\Omega}) + \epsilon^4f]. \end{aligned}$$

The stability properties of two inertial-acoustic modes, thermal and viscous modes can be obtained by analyzing the four kinds of solutions of the dispersion relation. The real parts of these solutions correspond to the growth rates of the perturbation modes and the imaginary parts correspond to their propagating properties. In following two sections we will numerically solve the dispersion relation according to the different disk structures. The stability of advection-dominated disks will be analyzed by assuming  $q \rightarrow 1$ . We note that the influence of radial viscous force on the disk stability has been investigated by some authors (Papaloizou & Stanley 1986; Wu, Yang & Yang 1994). Although it is included in our present study, more attentions in following sections will be paid on the effects of advection and thermal diffusion. In addition, we note that  $Re(\tilde{\sigma})$  is nearly in proportional to  $\alpha$  in all cases. Thus, the influence of different viscosity parameters on the



disk stability will not be detailed studied.

#### 4. Stability of optically thin disks

Previous results show that an optically thin disk is viscously stable but thermal unstable if it is dominated by local radiative cooling, and is both viscously and thermally stable if it is advection-dominated. In this section, we discuss its stability according to the different contribution of advection. Some parameters in the dispersion relation, such as  $x_i$  and  $y_i$ , are taken as the values given in Section 3 for an optically thin, gas pressure dominated disk. Others are dependent on the detailed disk structure or are set as the variables in following cases.

(a) *Geometrically thin disk without advection.* It is purely radiative cooling by thermal bremsstrahlung that balances the viscous dissipation, then  $q = 0$ . We take  $\tilde{\Omega} = \tilde{\chi} = 1$ ,  $\alpha = 0.01$ ,  $m = 0.01$ .  $\lambda/H$  is set from 1 to 80 for a geometrically thin disk according to the local restrictions ( Eq. (8)). By solving the dispersion relation, we get the results shown in Fig.1. In the long wavelength limit, the thermal mode is unstable and the viscous mode is marginally stable, which are in agreement with the previous results. However, when  $\lambda/H < 10$ , the thermal and viscous modes are stable and the acoustic instability becomes important. The thermal diffusion has nearly no effect on the disk stability in the long wavelength limit, but it stabilizes the acoustic modes and thermal mode significantly when the perturbation wavelength is shorter than  $20H$ .

(b) *Geometrically thin disk with very little advection.* The disk is dominated by radiative cooling but with very little advection, then we assume  $q = 0.01$ . Other parameters are the same as in case (a). In Fig. 2,  $m = 0.01$ , we see the thermal and viscous modes are nearly the same as in Fig. 1 but the stability properties of acoustic modes are quite

differen. If even very little advection is present, two acoustic modes are no longer complex conjugates as in the case without advection. The Inward propagating mode (I-mode) becomes stable while the outward propagating one (O-mode) becomes more unstable. In addition, the inclusion of thermal diffusion has the same effect as in case (a), which stabilizes the acoustic instability and thermal instability significantly especially in the short wavelength case. Moreover, we note the contribution of advection is associated with  $q/m$ , which can be clearly seen in the dispersion relation. It has nearly no effect on the thermal and viscous modes but has significant effects on two acoustic modes. In Fig. 3, we see the effect of advection increases as the increase of the value of  $q/m$ . If  $q/m < 0.1$ , however, this effect becomes negligible and the disk stability is nearly the same as in case (a).

(c) *Geometrically slim and advection-dominated disk.* In this case, the viscous dissipation is primarily balanced by advection and we take  $q = 0.99$ ,  $\alpha = 0.001$ ,  $H/r = 0.6$ . According to the local restrictions,  $\lambda/H$  is set from 0.02 to 2. If we choose  $\tilde{\Omega} = \tilde{\chi} = 1$  and  $m = 0.1$ , the solutions of the dispersion relation are shown in Fig. 4. We can clearly see that all four modes are all stable. The thermal diffusion tends to make the disk more stable. The O-mode and I-mode are not complex conjugates but the departure becomes less as the decrease of perturbation wavelength. Moreover, from Fig. 5 we can see, the departure of two acoustic modes becomes more less if  $q/m$  decreases. Such a departure is negligible if  $q/m < 0.3$ . But if  $q/m > 100$ , the O-mode can become unstable when  $\lambda/H > 0.8$ , even if the thermal diffusion is considered. In addition, we note that the optically thin, advection dominated disk is probably sub-Keplerian (Narayan & Yi 1994). Fig. 6 shows the solutions with  $\tilde{\Omega} = \tilde{\chi} = 0.01$ . In comparison with the case in Fig. 4, we note the viscous mode now becomes slightly unstable ( $Re(\tilde{\sigma})/\alpha \sim 0.1$ ) if without thermal diffusion. However, no viscous instability exists in a sub-Keplerian disk if the thermal diffusion is considered. The O-mode can becomes unstable only when  $q/m$  is very large,

such as  $q/m > 700$ . The change of  $q/m$  has significant effects on two acoustic modes, but has nearly no effects on the thermal and viscous modes.

## 5. Stability of optically thick disks

It is well known that an optically thick disk is thermally and viscously unstable if it is dominated by radiation pressure, and is thermally and viscously stable if it is dominated by gas pressure. If it is advection dominated, recent research suggested it is also both thermally and viscously stable. The local stability analyses of an optically thick disk without advection and thermal diffusion have been performed by Blumenthal et al. (1984) and Wu et al. (1995a, b). In this section, we will re-discuss its stability in detail according to the different disk structures and the contributions of advection and thermal diffusion. Some parameters in the dispersion relation, such as  $x_i$  and  $y_i$ , are taken as the values given in Section 3 for an optically thick disk.  $\tilde{\Omega} = \tilde{\chi} = 1$  is adopted through this section. Others are dependent on the detailed disk structure or are set as the variables.

(a) *Geometrically thin, gas pressure dominated disk without advection.* We take  $q = 0$ ,  $\alpha = 0.01$ ,  $m = 0.001$ .  $\lambda/H$  is set from 1 to 80. Fig. 7 shows the solutions. We see the thermal and viscous modes are stable but the acoustic modes are slightly unstable when  $\lambda/H > 7$ , if without the thermal diffusion. However, the acoustic modes become stable if the thermal diffusion is considered. In addition, we note that the thermal diffusion has nearly no effects on the thermal and viscous modes, especially when  $\lambda/H > 10$ .

(b) *Geometrically thin, gas pressure dominated disk with very little advection.* The disk is dominated by radiative cooling, so we take  $q = 0.01$ . Other variables have the same values as in case (a). From Fig. 8, we can clearly see that two acoustic modes now depart from each other due to the very little advection. The O-mode becomes more

unstable while the I-mode becomes stable. The inclusion of thermal diffusion does not change the instability of O-mode, although it decreases the growth rate of O-mode. The departure of two acoustic modes will become less as the decrease of  $q/m$ . The inclusion of thermal diffusion and the change of  $q/m$ , however, have nearly no effects on the thermal and viscous modes, which are always stable.

(c) *Geometrically thin, radiation pressure dominated disk without advection.* In this case, we take  $q = 0, \alpha = 0.01, m = 0.01$  and  $\lambda/H$  from 1 to 80. Fig. 9 shows the similar result as those in Blumenthal et al (1984) and Wu et al. (1995a). The inclusion of thermal diffusion decreases the growth rate of acoustic modes but has very little effects on the thermal and viscous modes. The thermal and viscous instabilities are dominant for long wavelength perturbations but the acoustic instability is dominant when  $\lambda/H < 20$ .

(d) *Geometrically thin, radiation pressure dominated disk with very little advection.* Fig. 10 shows the case similar as in Fig. 9 but with  $q = 0.01$ . In comparison with case (c), we see that the inclusion of very little advection leads to a significant departure of two acoustic modes but has nearly no effects on the thermal and viscous instabilities. The thermal diffusion tends to stabilize the acoustic modes but does not alter the instability of I-mode. The influence of advection term with different values of  $q/m$  on the acoustic modes is shown in Fig. 11. The decrease of  $q/m$  has significant effects to lessen both the instability of I-mode and the stability of O-mode by reducing the departure of two acoustic modes,.

(e) *Geometrically slim, radiation pressure and advection dominated disk.* In this case, we take  $q = 0.99, \alpha = 0.001, m = 0.1$  and  $\lambda/H$  from 0.02 to 2 for a slim disk according to the local restrictions. We note that there are no self-consistent solutions corresponding to two acoustic modes. Fig. 12 shows the results for thermal and viscous modes. We can clearly see that the thermal diffusion has a significant role to enhance the thermal

instability. The growth rate of thermal instability increases quickly as the decrease of perturbation wavelength. If without thermal diffusion, the thermal mode is only slightly unstable. The viscous mode, however, is always stable and does not change even if the thermal diffusion is included. In addition, we have also investigated the effects of the changes of  $q/m$ ,  $\alpha$  and the rotation law on the thermal and viscous modes. Those effects were found negligible. The reason why the acoustic modes disappears in our local analyses can be seen by investigating the dispersion relation (Eq. 14). In the limit when  $\alpha \sim 0$ , the dispersion relation becomes

$$y_3 \tilde{\sigma}^4 + [y_3 \tilde{\chi}^2 + (y_3 + x_1 y_1) \epsilon^2] \tilde{\sigma}^2 = 0. \quad (15)$$

Except two trivial solutions, other two solutions of above equation represent two propagating acoustic modes, which are described by

$$\tilde{\sigma} = \pm i [\tilde{\chi}^2 + (1 + \frac{x_1 y_1}{y_3}) \epsilon^2]^{1/2}, \quad (16)$$

where the positive imaginary part corresponds to the circle frequency of O-mode while the negative one corresponds to that of I-mode. The existence of self-consistent propagating acoustic modes requires the term in the square bracket of Eq. (16) is positive. For an optically thick, radiation pressure dominated disk,  $x_1 = -8$ ,  $y_1 = 4$  and  $y_3 = 12$ . The requirement above means  $\lambda/H > \sqrt{\frac{5}{3}} \frac{2\pi}{\tilde{\chi}}$ . If we choose  $\tilde{\chi} = \tilde{\Omega} = 1$ , the acoustic modes exist only when  $\lambda/H > 8$ , which is satisfied for a geometrically thin disk but not for a geometrically slim disk when a local analyses is presented. However, for a gas pressure dominated disk, where  $x_1 = y_1 = 1$  and  $y_3 = \frac{3}{2}$ , we can see from Eq. (16) that the acoustic modes always exist independent of the opacity and the geometry of the disk.

## 6. Discussion

We have performed detailed analyses to the local stability of accretion disks with advection. We found that for a geometrically thin, radiative cooling dominated disk, the presence of even very little advection has significant effects on the acoustic modes. Furthermore, those effects depend on what kinds of pressure is dominated in the disk. If the disk is gas pressure dominated, the presence of very little advection enhances the instability of O-mode and damps that of I-mode. But if it is radiation pressure dominated, the I-mode will become more unstable while the O-mode tends to become stable. Those effects are independent on the optical depth of the disk. The presence of very little advection has nearly no effect on the thermal and viscous modes. They are unstable in an optically thin disk and in an optically thick, radiation pressure dominated disk, but are stable in an optically thick, gas pressure dominated disk. These results are well agreement with those obtained in previous studies on the stability of a geometrically thin disk. In addition, we note that the inclusion of thermal diffusion has a significant effect to stabilize the acoustic modes in the short perturbation wavelength case, but has nearly no effects on the thermal and viscous modes especially in the long perturbation wavelength case. For a geometrically slim, advection-dominated disk, it is in general stable if it is optically thin. The viscous mode is slightly unstable in a sub-keplerian disk but will also become stable if the thermal diffusion is included. Only the O-mode can become unstable when  $q/m > 100$  even if the thermal diffusion is considered. If the disk is optically thick and advection-dominated, we found no self-consistent acoustic modes exist in our local analysis. The thermal mode is slightly unstable if without thermal diffusion, and it becomes much more unstable if the thermal diffusion is considered. The growth rate of thermal mode increases rapidly as the decrease of perturbation wavelength. It means the thermal instability is very important for an optically thick, advection-dominated disk, which was previously suggested to be thermally stable. It has been pointed out that the thermal instability of

an optically thick, advection-dominated disk is due to a large density change associated with a small pressure change (Kato et al. 1996), which is different from the case of a geometrically thin disk where there is no appreciable change of surface density.

The significant effects of very little advection on the acoustic modes is not surprised. The departure of O-mode and I-mode have been found by Chen & Taam (1993) and Wu & Yang (1994) when a casually limited viscosity is adopted. Together with the results in the present study, we think such a departure is resulted from the effects associated with the radial velocity. Due to the different disk structures, the influences of advection term on the acoustic modes in the disk with different kinds of dominant pressure are also different. In addition, we note the importance of thermal diffusion on the stability of advection-dominated disk has been pointed out by Kato et al. (1996). Our results prove that the thermal diffusion has a significant effects to damp the instability of an optically thin, advection dominated disk but enhance the thermal instability of an optically thick, advection-dominated disk. We can also see from Eq. (13) that the thermal diffusion terms is in proportional to  $(kH)^2$ , which can ne neglected for a geometrically thin disk but can not be neglected for a geometrically slim disk where  $kH \geq 1$ . Because the advection-dominated disk is usually not geometrically thin, the thermal diffusion must be included in a stability analysis. In addition, we note that the ratio of thermal discussion to viscous dissipation,  $Q_t/\Sigma\nu(r\frac{\partial\Omega}{\partial r})$ , is in proportional to  $(H/r)^2$ . It also means the thermal diffusion is not negligible when the structure of advection- dominated disk is constructed. From the well agreement of our results with some stability properties obtained from the analyses of  $\dot{M}(\Sigma)$  slope, we think the structure of an advection-dominated disk may be a little different if the thermal diffusion is included. Thus, a detailed calculation of disk structure with thermal diffusion is expected.

Finally, we would like to mention that our local stability analyses are performed under

some simplifications. Some physics processes neglected in our present study may not be negligible in some cases. For example, the two temperature effects in an optically thin disk may be very important if the disk temperature exceeds  $10^9\text{K}$  (Shapiro, Lightman & Eardley 1976). The viscous dissipation in radial direction is also neglected in this paper. In addition, we must note that the local stability may not be the same as that obtained from a global analysis. Moreover, the physics reason for the thermal instability in an optically thick, advection -dominated disk, which has been suggested by Kato et al. (1996), need to be confirmed in another detailed future work.

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## FIGURE CAPTIONS

**Figure 1.** The stability of an optically thin disk without advection. The solid, long dashed and short dashed lines correspond to acoustic modes, thermal and viscous modes respectively. Those lines with star centered represent the modes in the case where the thermal diffusion is considered.

**Figure 2.** The stability of an optically thin disk with very little advection. The solid and more short dashed lines correspond to the I-mode and O-mode. Others have the same meanings as in Fig. 1.

**Figure 3.** The influences of different  $q/m$  on the stability of acoustic modes in an optically thin disk with very little advection. The lines without any symbol centered, and those with open circle and with star centered correspond to  $q/m = 1, 5$  and  $0.1$  respectively.

**Figure 4.** The stability of an optically thin, advection-dominated disk. The solid, more short dashed, long dashed and short dashed lines correspond to I-mode, O-mode, thermal mode and viscous modes respectively. The lines with star centered correspond to those modes in the case with thermal diffusion considered.

**Figure 5.** The influences of different  $q/m$  on the stability of acoustic modes in an optically thin, advection-dominated disk. The lines without any symbol centered, and those with open circle and with star centered correspond to  $q/m = 10, 0.33$  and  $100$  respectively.

**Figure 6.** The same as Fig. 4 but for a sub-Keplerian disk where  $\tilde{\Omega} = \tilde{\chi} = 0.01$ . The solid and more short dashed lines with open circle centered represent respectively the I-mode and O-mode in the case with  $q/m = 1000$ .

**Figure 7.** The stability of an optically thick, gas pressure dominated disk without ad-

vection. The solid, long dashed and short dashed lines correspond to acoustic modes, thermal and viscous modes respectively. Those lines with star centered represent the case with thermal diffusion.

**Figure 8.** The stability of an optically thick, gas pressure dominated disk with very little advection. The solid and more short dashed lines with open circle centered correspond to I-mode and O-mode in the case with  $q/m = 1$ . Other lines have the same meanings as those in Fig. 4 and with  $q = 0.01$ ,  $m = 0.001$ .

**Figure 9** The stability of an optically thick, radiation pressure dominated disk without advection. The lines have the same meanings as in Fig. 7.

**Figure 10** The stability of an optically thick, radiation pressure dominated disk with very little advection. The lines have the same meanings as in Fig. 7.

**Figure 11** The influences of different  $q/m$  on the stability of acoustic modes in an optically thick, radiation pressure dominated disk with very little advection. The lines without any symbol centered, and those with open circle and with star centered correspond to  $q/m = 1$ , 0.2 and 2 respectively.

**Figure 12.** The stability of an optically thick, radiation pressure and advection dominated disk. The long dashed and short dashed lines correspond to the thermal mode and viscous mode. The lines with star centered represent the case with thermal diffusion.